



Fig. 1(3.11). CIE 1931  $(x, y)$ -chromaticity diagram showing isothermperature lines as computed by Kelly (1963).

reciprocal color temperature by nearly the same amount anywhere on the scale.

The term *correlated color temperature* is introduced when the chromaticity of a selective radiator (e.g., a fluorescent lamp) is not exactly equal to any of the chromaticities of a blackbody radiator. The correlated color temperature is defined as the temperature of the blackbody radiator whose perceived color most closely resembles that of the given selective radiator at the same brightness and under specified viewing conditions.

Judd (1936b) was the first to propose the term and compute so-called *isothermperature lines* for the evaluation of correlated color temperatures for selective radiators. If chromaticities are plotted in a uniform-chromaticity-scale diagram (see Section 6.4), isothermperature lines are short straight lines crossing the Planckian locus perpendicularly. In the CIE 1931  $(x, y)$ -chromaticity diagram these lines are, of course, no longer perpendicular to the Planckian locus; nevertheless, they facilitate the estimation of the correlated color temperature for any selective radiator with a chromaticity point somewhat off the Planckian locus.

Figure 1(3.11) shows Judd's isothermperature lines in the CIE 1931  $(x, y)$ -chromaticity diagram as computed by Kelly (1963) using the then valid value of Planck's radiation constant  $c_2 = 1.4380 \times 10^{-2} \text{ m} \cdot \text{K}$  and the CIE 1960  $(u, v)$ -diagram as the uniform-chromaticity-scale diagram.

Both Planck's radiation constant and the uniform-chromaticity-scale diagram have been changed since that time. The present value of  $c_2$  equals  $1.4388 \times 10^{-2} \text{ m} \cdot \text{K}$  (see Section 1.2.2), and the CIE 1960  $(u, v)$ -diagram has been superseded by the CIE 1976  $(u', v')$ -diagram. These changes forced the CIE to reconsider the specification of the isothermperature lines. It was decided that in calculating isothermperature lines, the present value of  $c_2$  be used, but the CIE 1960  $(u, v)$ -diagram be retained as the uniform-chromaticity-scale diagram. This decision has the effect that all correlated color temperature determinations of light sources made since 1968, that is, the year the present value of  $c_2$  was adopted, remain the same. A change from the CIE 1960  $(u, v)$ -diagram to the 1976 CIE  $(u', v')$ -diagram was considered undesirable as the resulting change in the correlated

color temperature scale would offer no advantage in practical applications. In this instance, continuity of practice was considered more important.

The change of  $c_2$  from  $1.4380 \times 10^{-2} \text{ m} \cdot \text{K}$  to  $1.4388 \times 10^{-2} \text{ m} \cdot \text{K}$  has only a small effect on the positioning of the isothermperature lines in Figure 1(3.11). In fact, on the scale of that figure, the change is of little or no consequence if the figure is used for the graphical determination of correlated color temperature.

In some practical applications, correlated color temperatures are required as part of a sequence of colorimetric calculations, for example, in the calculation of CIE daylight illuminants D (Section 3.3.4), and in the calculation of CIE color-rendering indices (Section 3.3.11). For such applications, numerical methods of determining correlated color temperatures are preferred over graphical methods (Mori et al., 1964; Robertson, 1968; Mahr et al., 1969).

Mori et al. (1964) propose the following method by means of which the slope of the isothermperature lines can be calculated.

The spectral radiant power  $S_T(\lambda)$  of a blackbody radiator at temperature  $T$  (in kelvin K) is obtained by an equation of the following form

$$S_T(\lambda) = c_1 \lambda^{-5} \left[ \exp\left(\frac{c_2}{\lambda T}\right) - 1 \right] \quad [4(3.11)]$$

which has been discussed in detail in Section 1.2.2 [see Eq. 6(1.2.2)]. The spectral radiant power distribution  $\{S_T(\lambda)d\lambda\}$  leads to the tristimulus values

$$\begin{aligned} U_T &= \int S_T(\lambda) \bar{u}(\lambda) d\lambda \\ V_T &= \int S_T(\lambda) \bar{v}(\lambda) d\lambda \quad [5(3.11)] \\ W_T &= \int S_T(\lambda) \bar{w}(\lambda) d\lambda \end{aligned}$$

where  $\bar{u}(\lambda)$ ,  $\bar{v}(\lambda)$ ,  $\bar{w}(\lambda)$  are linear combinations of the CIE 1931  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$  standard color-matching functions; that is,

$$\begin{aligned} \bar{u}(\lambda) &= \frac{2}{3}\bar{x}(\lambda) \\ \bar{v}(\lambda) &= \bar{y}(\lambda) \\ \bar{w}(\lambda) &= -\frac{1}{2}\bar{x}(\lambda) + \frac{3}{2}\bar{y}(\lambda) + \frac{1}{2}\bar{z}(\lambda) \end{aligned} \quad [6(3.11)]$$

Equations 6(3.11) define the tristimulus space in which the chromaticity diagram is the CIE 1960 UCS diagram required for the calculation of isothermperature lines.

From the  $(U_T, V_T, W_T)$ -tristimulus values, obtained by Eq. 5(3.11), the  $(u_T, v_T)$ -chromaticity coordinates are given by

$$\begin{aligned} u_T &= \frac{U_T}{U_T + V_T + W_T} \\ v_T &= \frac{V_T}{U_T + V_T + W_T} \quad [7(3.11)] \end{aligned}$$

The  $(u_T, v_T)$ -chromaticity points define the Planckian locus in the  $(u, v)$ -chromaticity diagram. The slope  $m_T$  of the tangent to the Planckian locus at the point  $(u_T, v_T)$  is given by the first derivative  $dv_T/du_T$ , that is,

$$m_T = \frac{dv_T}{du_T}$$

which is obtained from

$$\frac{dv_T}{du_T} = \frac{dv_T/dT}{du_T/dT} \quad [8(3.11)]$$

By using dots to denote derivatives with regard to temperature  $T$ , such as  $(dx_T/dT) \equiv \dot{x}_T$ , Eq. 8(3.11) then reads

$$\frac{dv_T}{du_T} = \frac{\dot{v}_T}{\dot{u}_T}$$

which can be expressed in terms of the tristimulus values  $U_T, V_T, W_T$  and their derivatives  $\dot{U}_T, \dot{V}_T, \dot{W}_T$  as follows:

$$\begin{aligned} m_T &= \frac{\dot{v}_T}{\dot{u}_T} \\ &= \frac{\dot{V}_T(U_T + V_T + W_T) - V_T(\dot{U}_T + \dot{V}_T + \dot{W}_T)}{\dot{U}_T(U_T + V_T + W_T) - U_T(\dot{U}_T + \dot{V}_T + \dot{W}_T)} \end{aligned} \quad [9(3.11)]$$

Whereas  $U_T, V_T, W_T$  are determined by Eq.

5(3.11),  $\dot{U}_T, \dot{V}_T, \dot{W}_T$  follow from

$$\begin{aligned}\dot{U}_T &= \int c_1 c_2 T^{-2} \lambda^{-6} \bar{u}(\lambda) \exp\left(\frac{c_2}{\lambda T_i}\right) \\ &\quad \cdot \left[ \exp\left(\frac{c_2}{\lambda T_i}\right) - 1 \right]^{-2} d\lambda \\ \dot{V}_T &= \int c_1 c_2 T^{-2} \lambda^{-6} \bar{v}(\lambda) \exp\left(\frac{c_2}{\lambda T_i}\right) \\ &\quad \cdot \left[ \exp\left(\frac{c_2}{\lambda T_i}\right) - 1 \right]^{-2} d\lambda \\ \dot{W}_T &= \int c_1 c_2 T^{-2} \lambda^{-6} \bar{w}(\lambda) \exp\left(\frac{c_2}{\lambda T_i}\right) \\ &\quad \cdot \left[ \exp\left(\frac{c_2}{\lambda T_i}\right) - 1 \right]^{-2} d\lambda \quad [10(3.11)]\end{aligned}$$

The isothermperature line passing through  $(u_T, v_T)$  is perpendicular to the tangent at that point of the Planckian locus; thus its slope  $t_T(u, v)$  is given by

$$t_T(u, v) = \frac{-1}{m_T} \quad [11(3.11)]$$

The integrations required in Eqs. 5(3.11) and 10(3.11) are, in practice, replaced by summations using the weighted-ordinate method from  $\lambda_a = 360$  nm to  $\lambda_b = 830$  nm with wavelength interval  $\Delta\lambda$  of constant 1-nm width (see Section 3.3.8).

The chromaticity points  $(u_T, v_T)$  of the Planckian locus and the slopes  $t_T(u, v)$  of the isothermperature lines passing through  $(u_T, v_T)$  can be transformed to the CIE 1931  $(x, y)$ -chromaticity diagram by means of the following equations:

$$\begin{aligned}x_T &= \frac{1.5u_T}{2 + u_T + 4v_T} \\ y_T &= \frac{v_T}{2 - u_T - 4v_T} \quad [12(3.11)]\end{aligned}$$

and

$$t_T(u, v) = \frac{2y_T[4t_T(u, v) - 1] + 2t_T(u, v)}{2x_T[4t_T(u, v) - 1] + 3}$$

Table 1(3.11) shows the results of the calculations made in accordance with Eqs. 4(3.11) to 12(3.11).

Robertson's (1968) method of determining the correlated color temperature of a source of given chromaticity is the numerical equivalent to the graphical method normally used, and involves an interpolation between two adjacent members of the set of 31 isothermperature lines defined in Table 1(3.11). With each line ( $i = 1$  to 31) is associated the correlated color temperature  $T_{ci}$ . A computer program is written that permits the search through the set of lines to find the two adjacent lines between which the chromaticity point of the given source lies. This is done by calculating in the  $(u, v)$ -chromaticity diagram the distance of the given chromaticity point from each of the 31 lines. If the coordinates of the chromaticity point of the given source are denoted by  $(u_s, v_s)$ , and  $(u_i, v_i)$  denotes the chromaticity point through which the  $i$ th isothermperature line of slope  $t_i$  passes, the required distance  $d_i$  is obtained from

$$d_i = \frac{(v_s - v_i) - t_i(u_s - u_i)}{(1 + t_i^2)^{1/2}} \quad [13(3.11)]$$

The chromaticity point  $(u_s, v_s)$  lies between the adjacent lines  $i = j$  and  $i = j + 1$  if the ratio  $d_j/d_{j+1}$  is negative.

The correlated color temperature  $T_c$  that corresponds to  $(u_s, v_s)$  is found by assuming that (a) the Planckian locus between  $T_j$  and  $T_{j+1}$  can be replaced by the arc of a circle whose center is at the intersection of the two adjacent isothermperature lines  $j$  and  $j + 1$  [see Figure 2(3.11)], and (b) the reciprocal color temperature is a linear function of distance along this arc. It follows that

$$T_c = \left[ \frac{1}{T_j} + \frac{\theta_1}{\theta_1 + \theta_2} \left( \frac{1}{T_{j+1}} - \frac{1}{T_j} \right) \right]^{-1} \quad [14(3.11)]$$

where  $\theta_1$  and  $\theta_2$  are the angles between the two isothermperature lines ( $T_j$  and  $T_{j+1}$ , respectively) and the line joining  $(u_s, v_s)$  to their intersection. For adjacent lines,  $\theta_1$  and  $\theta_2$  are small, and thus one can set  $\theta_1/\theta_2 = \sin \theta_1/\sin \theta_2$ , so that Eq. 14(3.11) becomes

$$T_c = \left[ \frac{1}{T_j} + \frac{d_j}{d_j - d_{j+1}} \left( \frac{1}{T_{j+1}} - \frac{1}{T_j} \right) \right]^{-1} \quad [15(3.11)]$$

**Table 1(3.11) Isotemperature Lines ( $c_2 = 1.4388 \times 10^{-2} \text{ m} \cdot \text{K}$ ) Calculated in Accordance with the Method Proposed by Mori et al. (1968)**

<i>i</i>	Reciprocal Megakelvin ( $\text{MK}^{-1}$ )	Temperature <i>T</i> (K)	Chromaticity Coordinates in CIE 1960 UCS Diagram			Chromaticity Coordinates in CIE 1931 Diagram		
			$u_T$	$v_T$	Slope $t_T(u, v)$	$x_T$	$y_T$	Slope $t_T(x, y)$
1	0	$\infty$	0.18006	0.26352	-0.24341	0.23987	0.23404	-0.68705
2	10	100,000	0.18066	0.26589	-0.25479	0.24258	0.23802	-0.72797
3	20	50,000	0.18133	0.26846	-0.26876	0.24560	0.24240	-0.77926
4	30	33,333	0.18208	0.27119	-0.28539	0.24890	0.24714	-0.84248
5	40	25,000	0.18293	0.27407	-0.30470	0.25251	0.25222	-0.91976
6	50	20,000	0.18388	0.27709	-0.32675	0.25645	0.25763	-1.01403
7	60	16,667	0.18494	0.28021	-0.35156	0.26070	0.26333	-1.12891
8	70	14,286	0.18611	0.28342	-0.37915	0.26526	0.26930	-1.26959
9	80	12,500	0.18740	0.28668	-0.40955	0.27011	0.27547	-1.44313
10	90	11,111	0.18880	0.28997	-0.44278	0.27524	0.28182	-1.65981
11	100	10,000	0.19032	0.29326	-0.47888	0.28063	0.28828	-1.93507
12	125	8,000	0.19462	0.30141	-0.58204	0.29518	0.30477	-3.08425
13	150	6,667	0.19962	0.30921	-0.70471	0.31101	0.32116	-6.18336
14	175	5,714	0.20525	0.31647	-0.84901	0.32775	0.33690	-39.34888
15	200	5,000	0.21142	0.32312	-1.0182	0.34510	0.35162	11.17883
16	225	4,444	0.21807	0.32909	-1.2168	0.36276	0.36496	5.34398
17	250	4,000	0.22511	0.33439	-1.4512	0.38045	0.37676	3.68730
18	275	3,636	0.23247	0.33904	-1.7298	0.39792	0.38690	2.90309
19	300	3,333	0.24010	0.34308	-2.0637	0.41502	0.39535	2.44455
20	325	3,077	0.24702	0.34655	-2.4681	0.43156	0.40216	2.14300
21	350	2,857	0.25591	0.34951	-2.9641	0.47464	0.40742	1.92863
22	375	2,677	0.26400	0.35200	-3.5814	0.46262	0.41121	1.76811
23	400	2,500	0.27218	0.35407	-4.3633	0.47701	0.41368	1.64291
24	425	2,353	0.28039	0.35577	-5.3762	0.49059	0.41498	1.54240
25	450	2,222	0.28863	0.35714	-6.7262	0.50338	0.41525	1.45962
26	475	2,105	0.29685	0.35823	-8.5955	0.51541	0.41465	1.39021
27	500	2,000	0.30505	0.35907	-11.324	0.52669	0.41331	1.33101
28	525	1,905	0.31320	0.35968	-15.628	0.53723	0.41131	1.27989
29	550	1,818	0.32129	0.36011	-23.325	0.54712	0.40882	1.23522
30	575	1,739	0.32931	0.36038	-40.770	0.55640	0.40593	1.19579
31	600	1,667	0.33724	0.36051	-116.45	0.56508	0.40271	1.16074

<sup>a</sup>Data supplied in part by A. R. Robertson, private communication, 1980.

The success of the method depends on choosing a set of isotemperature lines with sufficiently narrow spacing so that the errors introduced by the above assumptions are negligible. Robertson (1968) has shown that the set given in Table 1(3.11) leads to very satisfactory results.

3.12 COLORIMETRIC INSTRUMENTATION

Modern colorimetric instrumentation is designed to provide automatically the tristimulus values

and chromaticity coordinates of a given color stimulus without making use of an actual human eye as a measuring component of the instrument. There are three kinds of colorimetric instrumentation: the *spectroradiometer*, the *spectrophotometer*, and the *tristimulus-filter colorimeter*. Each kind is commercially available in a variety of makes, degrees of sophistication, and specializations to suit specific colorimetric applications. It is not the purpose of this book to treat in detail colorimetric instrumentation and to describe details of their design and operation. Such information is readily