CHAPTER VIII

THE SPECTRUM

1. The line spectrum

The lines radiated by the high pressure mercury vapour discharge are all broadened, the effective width increasing with the pressure and the current density. In III,5 the difference of the broadening of the 5770 Å line and the 10140 Å line was used to determine the temperature. At the same time a shifting of the lines towards shorter or towards longer wavelengths occurs. Broadening and shifting of a line are caused by broadening and difference in displacement of the initial and the final levels. In fig. 57 we reproduce

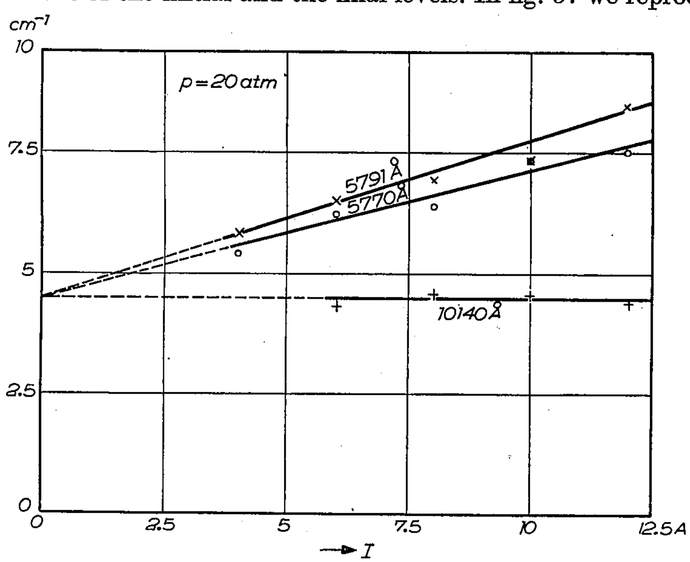

Fig. 57. Shifting to violet as a function of the current at 20 atm [117]

a figure by Schulz [117] representing the shifting in the direction of shorter wavelengths as a function of the current for a pressure

of 20 atm. Figure 57 is quite analogous to fig. 20, where the width of these lines is given. Most of the lines shift to longer wavelengths. Fig. 60 shows the increasing displacement of the green line 5461 Å with increasing density; as is to be seen, the displacements are considerable at high pressures. At $E = 1000$ V/cm ($m \approx 10$ mg/cm; $p \approx 300$ atm) the 5461 Å line lies at 5500 Å, so that the displacement amounts to 130 cm^{-1}.

Besides the wavelength and the width of a line, the intensity of the line is important. The intensity of each line is a function of P, m and d. The intensity of the yellow lines of discharge D has been used already in III,6.

In table 9 the intensity is given of the lines and of the continuous spectrum of the U.V. standard [84]. This is a high pressure mercury vapour discharge in a quartz tube of 1.8 cm inner diameter, with an arc length of 10.5 cm, the ends of which are covered, whereas the central part (8 cm) radiates. The lamp is filled with 1.0 cm A and a quantity of mercury such that the arc voltage is 130 V when the mercury is completely vaporised ($m \approx 3.7$ mg/cm). The input is 250 watts d.c. With these data the discharge is defined (this is not strictly true since the electrode losses may vary slightly from one lamp to another; since, however, highly emissive electrodes are used the electrode losses are small, so that such variations have little effect on the total radiation). It would be better to define the U.V. standard by d, m and P, instead of d, E and P, because in the latter case a small change in d affects the temperature of the discharge and also as a result the energy distribution in the spectrum.

Assuming an electrode loss of 20 watts, the energy consumption of the radiating 8 cm is $230 \times 8/10.5 = 175$ watts. From these 175 W, 80 watts are lost by thermal conduction and 72 % of the remainder, (namely $68\frac{1}{2}$ watts) are measured as radiated energy (see eq. 2. 3.4). The other $26\frac{1}{2}$ watts are radiated by the discharge but are absorbed in the cylindrical non-luminous zone, the quartz glass tube and the air.

The meaurements of table 9 were carried out without forming an image of the discharge on the slit of the monochromator, because

TABLE 9

Absolute intensity of the radiation of 8 cm arc length of the U.V. standard ($d = 1.8$ cm, $m \approx 3.7$ mg/cm, $P = 22$ W/cm)

Wavelength (in Å)	Energy in the lines (watt)			Energy (watt) in the continuous spectrum per 100 Å		
	Fischer [48]	Rössler [108]	Van Stekelenburg [125]	Fischer [48]	Rössler [108]	Van Stekelenburg [125]
> 5791		7.1			5 (total)	
5770/91		5.55	4.5		0.050	0.043
5461		6.78	5.5		0.040	0.043
4916		0.07	0.05		0.050	0.044
4358	6.85	5.45	4.5	0.092	0.065	0.058
4047/78	4.04	3.59	2.75	0.0575	0.080	0.060
3906	0.11	0.06	0.06	0.12	0.10	0.067
3650/63	9.0	7.53	7.1	0.40	0.27	0.11
3341	0.855	0.55	0.58	0.77	0.48	0.285
3126/32	6.4	5.05	5.1	0.52	0.58	0.30
3022/26	2.96	2.15	2.25	0.30	0.59	0.30
2967	1.66	1.15	1.36	0.39	0.44	0.30
2925	0.21	0.11	0.13	0.23	0.41	0.30
2894	0.65	0.44	0.48	0.36	0.41	0.30
2804	1.12	0.77	0.90	0.315	0.465	0.31
2753/60	0.36	0.29	0.30	0.28	0.57	0.34
2699	0.43	0.34	0.34	0.51	0.82	0.37
2640/52	2.22	1.80	1.72			
< 2640	7.5	2			6 (total)	
		51 W			17 W	

the intensity distribution over the cross section varies for different lines (see II,2). The total energies in table 9 are derived from the measurements in a direction perpendicular to the axis of the tube by using the ratio 0.86 for the mean spherical intensity divided by the intensity perpendicular to the axis [108]. As table 9 shows there are considerable differences in the absolute intensities as given by various authors. The total radiation $51 + 17 = 68$ watts found by Rössler agrees very well with the total radiation of $68\frac{1}{2}$ watts as measured with a thermopile.

We shall not list here the intensities of the lines for other discharges, but we shall indicate how the intensity ratios change with varying P, m and d. This may be done by bearing in mind that the intensity of a line depends on:

1. the number of atoms excited to the initial level of the line,
2. the transition probability of the line,
3. the self-absorption of the line.

The absorption of the tube wall will not be considered here (for a quartz glass tube the absorption is small between 2000 Å and 4 μ). The self-absorption of a line in the discharge is determined by the concentration of atoms in the final level, as a function of r. The variation of the absorption is the uncertain factor, but as the absorption is small except in the case of the resonance lines, we shall ignore these variations.

Since the transition probability is a property of the atoms for the transition considered, the variation of the intensity of a line is determined by the variation of the number of atoms in the initial level. According to (1. 3.1) the concentration of atoms excited to the level V_k above the ground level is given by $n_0(g_k/g_0)$ exp $(-eV_k/kT)$. By calculating with an effective width and an effective temperatu reof the discharge (see IV,4) the intensity of a line is proportional to m exp $(-eV_k/kT_{\text{eff}})$. From this we may derive the variation of the intensity ratios if we know how T_{eff} varies. This variation is given by equations (4. 4.4) and (4. 4.5). These equations are, however, derived on the assumption that the fraction of the atoms within the effective radius is always the same. Generally, the effective diameter is not important, because if this width is taken somewhat larger, and the temperature is chosen correspondingly lower, the absolute value of the total radiation and the potential gradient do not vary much. The intensity *ratios* will, however, depend on a variation of the effective width. We may thus expect some deviations from the calculated intensity variations.

At constant m and d and varying P the total radiation is proportional to (P—10). If the temperature remained constant, this would mean that the intensity of every line would increase proportionally with (P—10). According to Kern and Schulz [74] the diameter of the discharge increases approximately proportionally with $P^{1/2}$, T remaining indeed constant. However, according to measurements of Kenty and Karash [69] and Koch [80] the axis temperature increases slowly with increasing power input. This is what is to be expected from (4. 4.5). The lines with high initial levels will therefore increase in intensity a little more rapidly than (P—10) and the lines with low initial levels a little less rapidly than (P—10). To a first approximation, however, we expect for every line an increase proportional to (P—10). In fig. 46 the intensity of the yellow lines is seen to be proportional to (P—P_{cond}—P_{cr}), P_{cond} being the conduction loss in pure mercury vapour (10 W/cm) and P_{cr} the additional loss due to the rare gas. In fig. 58 the intensity of different lines is drawn as a function

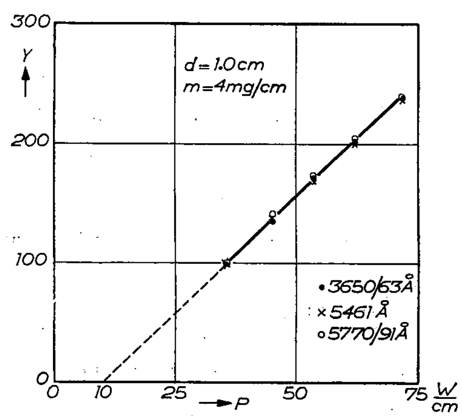

Fig. 58. Intensity variation as a function of P at constant d and m [74]. The ordinate y represents the intensity of the lines in percentages of the intensity at 36 W/cm

of P according to measurements by Kern and Schulz [74], showing equal increase for different lines.

At constant P and d, the effective temperature decreases with increasing m. We thus expect that with increasing m the intensity of lines with the higher initial levels will decrease, and that the intensity of lines with the lower initial levels will increase. This is indeed the case. Fig. 59 shows the intensity variation as a function of m for $d = 1.0$ cm and $P = 60$ W/cm, after measurements by Kern and Schulz [74], who measured the intensity as a function

of E. The lines 3650/63 Å and 5770/91 Å, with initial levels at 8.85 V, decrease with increasing m, whereas the lines 4358 Å and 5461 Å, with initial levels at 7.73 V, increase.

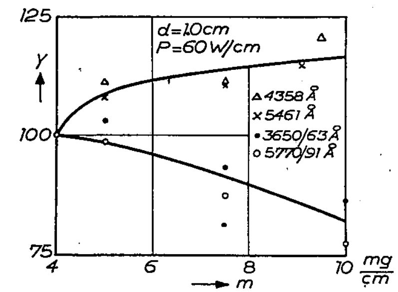

Fig. 59. Intensity variation as a function of m at constant d and P, derived from measurements as a function of E [74]. The ordinate y represents the energy of the lines in percentages of the energy at $m = 4$

The curves, of fig. 59 are, however, not exactly what one would expect. Firstly, we may expect that the intensities of lines with initial levels at about the same height as the fictitious mean upper level (7.8 V — see II,4), would be independent of T (P = constant). For the 4358 Å line and the 5461 Å line one would, for this reason, expect a horizontal line in fig. 59. Secondly, the intensity of the continuous spectrum increases considerably between $m/d^2 = 4$ and $m/d^2 = 10$. The total energy in the line spectrum therefore diminishes between $m/d^2 = 4$ and $m/d^2 = 10$ (see equation (8. 5.12)). Thus we expect the curve of fig. 59 for the 4358 Å and 4561 Å line to decrease somewhat at increasing m. That this is not the case is probably a result of the method by which the intensity of the lines was measured, i.e. with filters. In this method a part of the continuous energy is necessarily included in the measurement, and since the continuous energy is increasing considerably between $m/d^2 = 4$ and $m/d^2 = 10$ the measured increase may be due to the increase of the continuous energy.

The difference in the two curves of fig. 59 is caused by the difference in the height of the initial levels (8.85 and 7.73 V respectively) and by the decreasing temperature. According to (8. 6.1), the temperature between $m = 4$ and $m = 10$ decreases from 6050 to 5500° K. This causes a change in the ratio of the population of the two initial levels of exp $[11600 \times (8.85 - 7.73)/ /5500 - 11600 \times (8.85 - 7.73)/6050] = \exp(0.21) = 1.23$. In fig. 59 the change amounts to $1.16/0.82 = 1.4$. The calculated change is thus smaller than the measured one, despite the large temperature decrease (partly caused by the increase of the continuous intensity). This may be the result of a different percentage of continuous energy measured together with the lines, or of a variation of the absorption of the 4358 Å and 5461 Å line (the

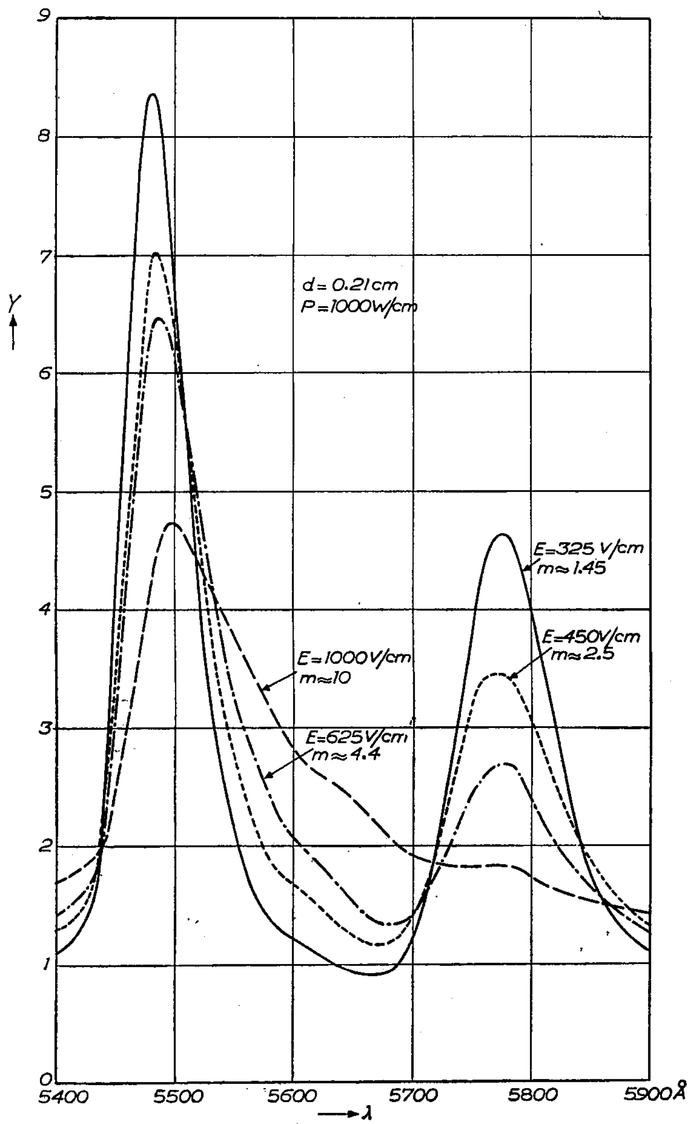

Fig. 60. The energy distribution of watercooled lamps at constant d (0.21 cm) and P (1000 W/cm) and varying E. $y d\lambda$ represents the energy radiated in the range λ, $\lambda + d\lambda$ (scale arbitrary, but the curves are mutually comparable). At $E = 1000$ V/cm ($p \approx 300$ atm) the yellow lines have practically disappeared

absorption of the 3650/63 and 5770/91 lines being small). With temperatures of 6050 and 5500° K, the intensity variation of the 3650/63 and 5770/91 Å lines between $m = 4$ and $m = 10$ is calculated as 4 exp (— 102500/6050) : : 10 exp (— 102500/5500) = 100 : 45. The measured ratio of 100 : 82 is, as explained above, probably caused by the insufficient separation of line energy and continuous energy, the latter increasing with increasing m.

Discharges with constant P and m and varying d are similar, so that the intensity ratios should be independent of d. For a given value of m this is only true in a certain range of diameters, at the lower limit the continuous spectrum becoming important, causing a decrease of the temperature (see VIII,6), and at the higher limit the discharge ceasing to be a high pressure discharge.

The line spectrum becomes relatively less important at higher pressures where the continuous spectrum predominates, (see VIII,5). Moreover, another phenomenon occurs at very high pressures: lines with high initial levels disappear entirely. In fig. 60 the intensity as a function of wavelength at constant P and d and variable E is reproduced (unpublished measurements of the author). The yellow lines have practically disappeared at $E = 1000$ V/cm ($p \approx 300$ atm). This is caused by the broadening of the levels with increasing pressure and current density. Near the ionization potential the levels are lying close together so that the levels overlap as their broadening increases, causing a decrease of the ionization potential. With increasing pressure this continuous energy band becomes more extensive. If the band reaches the initial level of a line, the line disappears into the continuous background. This lowering of the ionization potential will be discussed further in VIII,8.

2. The continuous spectrum in the neighbourhood of the resonance lines

The broadening of the spectral lines considered in III,5 was a symmetrical broadening, caused by collisions of the excited atom with normal atoms, electrons or ions. The broadening of the resonance lines is unsymmetrical and may be treated on a statistical basis. In fig. 61 curve a represents the height of the initial level of the resonance line as a function of the distance between the excited atom and an atom in the ground state, whereas curve b applies to two unexcited atoms. If no other atom is in the neighbourhood

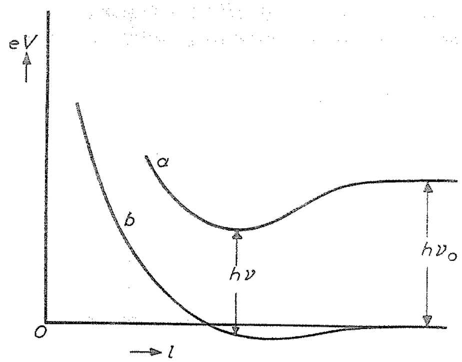

Fig. 61. Potential curve for an excited atom (a) and a normal atom (b)

of the excited atom (l large) a quantum of frequency ν_0 is emitted. However, if a second atom is present at a distance l, the radiated frequency corresponds to the vertical distance between the curves a and b at l. Since the distance between the curves diminishes with decreasing l, the emitted frequency also decreases. At low vapour pressures the distances between the atoms are so large that only the frequency ν_0 is radiated. With increasing density, however, the probability of a transition when other atoms are in the neighbourhood increases. As transitions occur at different distances, we get, instead of one frequency ν_0, a continuous range of frequencies from ν_0 to smaller values, the band being broader for higher densities.

Fig. 61 holds for both the emission and the absorption. The emission band of the central core may thus be absorbed in the cylindrical non-luminous zone. The absorption is high, because in this zone the density is very large as a result of the lower temperature (see fig. 26).

Fig. 62 shows the ultraviolet region of the spectrum as produced by various mercury vapour discharges. The absorption gap extending from the resonance line 2537 Å to longer wavelengths

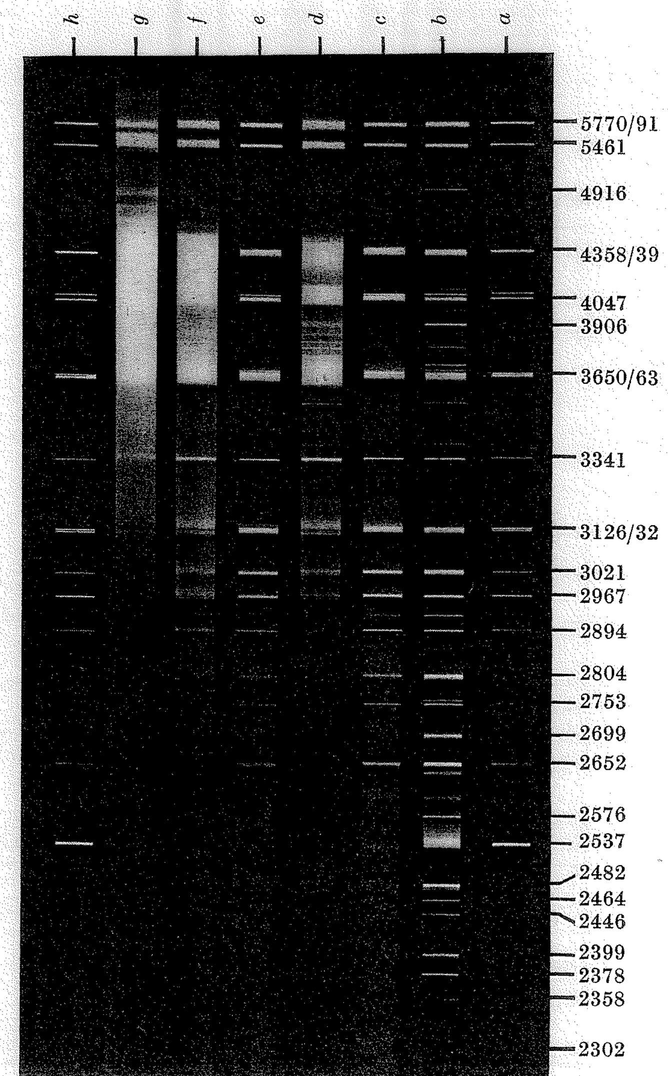

Fig. 62. The ultraviolet part of the spectrum for various discharges.

a. low pressure mercury vapour discharge – b. high pressure mercury vapour discharge of the 1 atm type ($\phi = 16$ mm, $E = 9$ V/cm, $P = 22\frac{1}{2}$ W/cm) – c. high pressure mercury vapour discharge at higher pressure ($\phi = 4$ mm, $E = 100$ V/cm, $P = 50$ W/cm, $p \approx 18$ atm) – d. compact source lamp ($\phi = 30$ mm, $E = 150$ V/cm, $P = 1300$ W/cm, $p \approx 45$ atm) – e. watercooled high pressure mercury vapour discharge of relatively low pressure ($\phi = 4$ mm, $E = 110$ V/cm, $P = 400$ W/cm, $p \approx 15$ atm) – f. normal watercooled high pressure mercury vapour discharge ($\phi = 2$ mm, $E = 400$ V/cm, $P = 800$ W/cm, $p \approx 100$ atm) – g. watercooled high pressure mercury vapour discharge ($\phi = 1$ mm, $E = 1000$ V/cm, $P = 800$ W/cm, $p \approx 200$ atm) – h. low pressure mercury vapour discharge

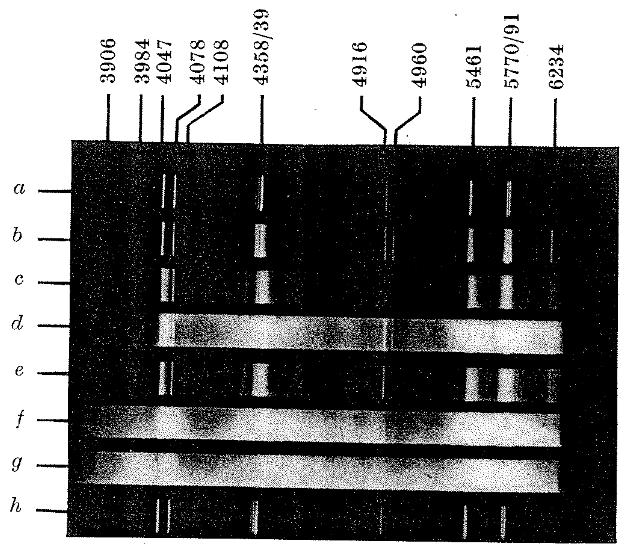

Fig. 68. The visible part of the spectrum

a. low pressure mercury vapour discharge
b. high pressure mercury vapour discharge of the 1 atm type ($\phi = 16$ mm, $E = 9$ V/cm, $P = 22\frac{1}{2}$ W/cm)
c. high pressure mercury vapour discharge at higher pressure ($\phi = 4$ mm, $E = 100$ V/cm, $P = 50$ W/cm, $p \approx 18$ atm
d. compact source lamp ($\phi = 30$ mm, $E = 150$ V/cm, $P = 1300$ W/cm, $p \approx 45$ atm)
e. watercooled high pressure mercury vapour discharge of relatively low pressure ($\phi = 4$ mm, $E = 110$ V/cm, $P = 400$ W/cm, $p \approx 15$ atm)
f. normal watercooled high pressure mercury vapour discharge ($\phi = 2$ mm, $E = 400$ V/cm, $P = 800$ W/cm, $p \approx 100$ atm)
g. watercooled high pressure mercury vapour discharge ($\phi = 1$ mm, $E = 1000$ V/cm, $P = 800$ W/cm, $p \approx 200$ atm)
h. low pressure mercury vapour discharge

is clearly visible in *c*, *d*, *f* and *g*. From the resonance line 1850 Å another absorption gap extends to longer wavelengths. These gaps broaden with increasing pressure and tube diameter, so that at very high pressures the gap of the 1850 Å line extends to 2537 Å (fig. 62 *g*) and no radiation of a shorter wavelength than about 2800 Å can escape from the lamp.
De Groot [59] was able to show that the width of the absorption gap as a function of the pressure agrees with what is to be expected from the nature of the curves *a* and *b* in fig. 61 ($\nu_0 - \nu = C/l^6$).

3. The continuous spectrum due to recombination

A continuous background is observed not only in the neighbourhood of the resonance lines, but throughout the whole spectrum. Unsöld [127] has calculated the intensity of that part of the continuous spectrum which is due to recombination of electrons with positive ions. He derives a radiation per cm³ per unit frequency range of

$$P^*_{\text{rec}} = \psi \frac{128\,\pi^3}{3\sqrt{3}}\, 10^{-7} \left(\frac{e^2}{hc}\right)^3 n_0\, kT \exp\{-e(V_{\text{i}} - \Delta V_{\text{i}})/kT\} =$$
$$= 1.2 \times 10^{-7}\, \psi\, p \exp\{-e(V_{\text{i}} - \Delta V_{\text{i}})/kT\}, \qquad (8.\ 3.1)^*$$

(P^*_{rec} in W/cm³, p in atm). The weight factor ψ is 4 in the case of mercury, and ΔV_{i} is the diminution of the ionization potential occurring at high pressures, which has been mentioned in VIII,1. Apart from the numerical factor one may easily understand why (8. 3.1) has this form, because the recombination probability is proportional to the product of the electron- and the ion-concentration $n_{\text{i}} n_{\text{e}}$. According to (1. 3.2) $n_{\text{i}} n_{\text{e}}$ is proportional to n_0 $\exp\{-e(V_{\text{i}} - \Delta V_{\text{i}})/kT\}$ and this factor occurs in (8. 3.1).
Unsöld thus obtains a constant intensity per unit of frequency. For the discharge D for which T_{e} and n_0 are known as a function of r and the pressure is such that $\Delta V_{\text{i}} = 0$, we may calculate the intensity of the continuous spectrum with (8. 3.1) and compare the result with the measured intensity. The energy in the con-

tinuous spectrum per unit of frequency and per cm of arc length is, according to (8. 3.1) with $p = 0.88$ if $\Delta V_i = 0$ (P_{rec} in W/cm):

$$P_{rec} = \int_0^R 2\pi r P^*_{rec}\, dr = 4.8 \times 10^{-7} \times 0.88\, \pi \int_0^{R^2} \exp(-121000/T_e)\, dr^2. \quad (8.3.2)^*$$

In fig. 63 we have plotted exp ($-$ 121000/T/) for the discharge D as a function of r^2. The area below the curve amounts to 2.85 10^{-10}, resulting in a radiation:

$$P_{rec} = 4.8 \times 0.88 \times \pi \times 2.85 \; 10^{-17} = 3.75 \; 10^{-16} \text{ watt} \qquad (8.\,3.3)$$

per cm of arc length and per unit of frequency.
The energy emitted by the continuous spectrum of the discharge D

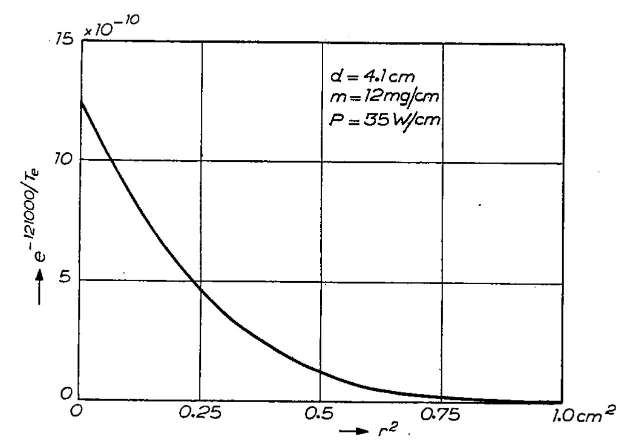

Fig. 63. Determination of the integral of (8. 3.2) for the discharge D

has also been measured. In the part of the spectrum between 7000 Å and 4000 Å a total continuum energy of 0.6 W per cm of arc length was found. As the number of frequency units in this part is 3.2 10^{14}, the calculation leads according to (8. 3.3) to 3.2 $10^{14} \times 3.75 \times 10^{-16}$ watt = 0.12 watt. Thus experimentally we find a continuous radiation which is 5 times stronger than expected from equation (8. 3.1). Therefore, either the recombination continuum is stronger than calculated by Unsöld, or

there is still another mechanism which produces the continuous spectrum. The latter seems to be probable because the intensity of the recombination continuum according to (8. 3.1) is proportional to the pressure, in the same way as the line spectrum. With increasing pressure, the ratio of the continuous energy to the line energy would thus to a first approximation remain the same. Because of the decrease of the temperature with increasing pressure, the continuous energy would even show a relative decrease, since the ionization energy is higher than the excitation energy of the levels [1]). With increasing pressure, however, the continuous energy increases relative to the energy in the lines. Thus the origin of at least a part of the continuous spectrum must be different from that assumed above.

4. THE CONTINUOUS SPECTRUM DUE TO MOLECULAR RADIATION

Rössler [109, 110] has determined the height of the initial level of the continuous spectrum by comparing the rate of decrease, from the axis to the wall, of the continuous intensity [2]) and of the intensity of lines with initial levels at different heights. As shown in II,2, lines with higher initial levels decrease in intensity more rapidly, so that the rate of decrease is a measure of the height of the initial level. In this manner Rössler found a value of 8.6 volts for the height of the initial level of the continuous spectrum. In the case when the continuous spectrum is mainly caused by recombination, we have from (8. 3.1), to expect a value of about 10 V ($\approx V_i - \Delta V_i$). Thus Rössler concludes that the recombination

[1]) The decrease of V_i with increasing pressure causes a relative increase of the recombination continuum. As shown by fig. 70 the decrease of V_i beginns at $m/d^2 \approx 3$, whereas according to fig. 64 the increase of the continuous intensity is also present at $m/d^2 < 3$. Therefore, the decrease of V_i with increasing pressure cannot be the only reason for the relative increase of the continuous intensity.

[2]) The intensity decrease was measured in the infrared and visible part of the spectrum. In the ultraviolet, Rössler [107] finds, after an initial decrease, an increase of the intensity towards the wall, which is explained as caused by fluorescence of the vapour in the non-luminous zone excited by the resonance radiation from the core.

is not the main cause of the continuum and he suggests that the continuous spectrum is caused by the collision of an excited atom with a normal atom. The intensity of this continuous spectrum per cm of arc length becomes:

$$P_{\mathrm{mol}} = C_{\mathrm{m}}\, p^2\, T_{\mathrm{eff}}^{-3/2}\, \pi\, \beta^2 R^2 \exp\left(-e V_{\mathrm{m}}/kT_{\mathrm{eff}}\right) \qquad (8.\,4.1)$$

with $V_{\mathrm{m}} = 8.6$ volts and $\beta = 0.42$ (see IV,4). C_{m} is a proportionality factor, which depends on the wavelength and according to Rompe and Steenbeck [103] is very small, but which according to Rössler might be considerably larger, because of a flat minimum in the potential curve of the system: excited atom-normal atom. C_{m} thus not being known, we cannot check equation (8. 4.1) by comparing it with the absolute value of the measured intensity. The intensity of this continuous spectrum is, however, proportional to p^2, so that at increasing pressure the intensity of the continuum indeed increases with respect to the intensity of the lines, the latter increasing proportionally to p.

5. The Intensity of the Continuous Spectrum as a Function of the Pressure

In order to decide to what extent the recombination continuum and the molecular continuum contribute to the measured intensity, the intensity of the continuous spectrum has been measured as a function of the pressure, together with the intensity of the yellow lines, both on a relative scale [37]. The intensity of the continuous spectrum was assumed to be proportional to the mean of the intensity at 6400 Å and 4800 Å. It was found that, for a given tube, the ratio of the intensity of the continuous spectrum P_{cont} to the intensity of the yelllow lines P_{5780} was independent of the power, as table 10 shows. This proves that the heights of the initial levels of both radiations are not very different. Thus we obtain for the height of the initial level of the continuous spectrum a value of about 8.85 V, which is in accordance with the value of 8.6 V found by Rössler.

In analogy with table 10 we may now determine the mean value of $P_{\mathrm{cont}}/P_{5780}$ for a number of different combinations of d and m.

TABLE 10

Intensities in arbitrary units, at various inputs, for a discharge with $d = 1.38$ cm, $m = 6$ mg/cm

P (W/cm)	P_{cont}	P_{5780}	$P_{\mathrm{cont}}/P_{5780}$
21.5	0.145	0.175	0.83
32.3	0.275	0.32	0.86
43.9	0.415	0.47	0.88[5]
54.8	0.530	0.66	0.80
67	0.705	0.81	0.87
			0.85

In fig. 64 we have plotted these ratios as a function of m/d^2. For $m/d^2 < 7$ (corresponding to pressures below about 9 atm) the points lie on a straight line intersecting the $P_{\mathrm{cont}}/P_{5780}$ axis at a positive value. This can be explained as follows: The intensity

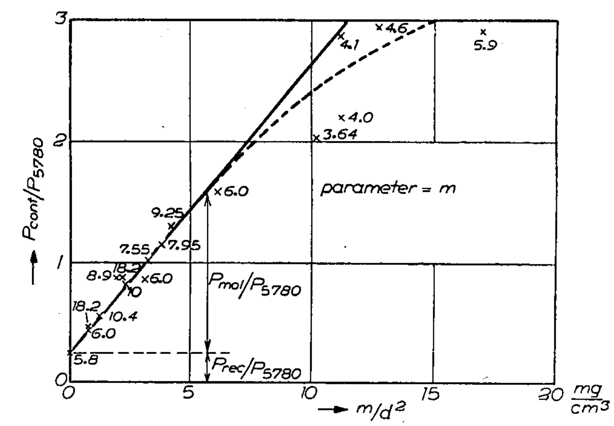

Fig. 64. The ratio of the continuous intensity to the intensity of the yellow lines (arbitrary scale) as a function of m/d^2 [37]

P_{cont} of the continuous spectrum is the sum of the recombination and the molecular intensities, or

$$\left.\begin{aligned} P_{\mathrm{cont}} = C_{\mathrm{r}}\, p\, \beta^2 R^2 \exp\{-e(V_{\mathrm{i}} - \Delta V_{\mathrm{i}})/kT_{\mathrm{eff}}\} + \\ + C_{\mathrm{m}}\, p^2\, T^{-3/2} \beta^2 R^2 \exp\left(-e V_{\mathrm{m}}/kT_{\mathrm{eff}}\right). \end{aligned}\right\} \qquad (8.\,5.1)$$

The intensity of the yellow lines is

$$P_{5780} = C_{5780}\, p\, T^{-1} \beta^2 R^2 \exp(-eV_k/kT_{\text{eff}}), \qquad (8.\ 5.2)$$

with $V_k = 8.85$ volts.
Neglecting the variation of the powers of T compared with the variation of the exponential functions, we thus obtain the ratio

$$P_{\text{cont}}/P_{5780} = \frac{C_r \exp\{-e(V_i - \Delta V_i)/kT_{\text{eff}}\} + C_m p \exp(-eV_m/kT_{\text{eff}})}{C_{5780} \exp(-eV_k/kT_{\text{eff}})}. \qquad (8.\ 5.3)$$

Since V_k and V_m are about the same (8.85 V and 8.6 V resp.), and $(V_i - \Delta V_i)$ is not very much larger, we may write approximately with $p \backsim m/d^2$:

$$P_{\text{cont}}/P_{5780} = \frac{C'_r + C'_m\, m/d^2}{C'_{5780}}. \qquad (8.\ 5.4)$$

According to (8. 5.4), P_{cont}/P_{5780} is a linear function of m/d^2. This is for $m/d^2 < 7$ in agreement with fig. 64. Thus we interpret the part cut off from the ordinate axis as due to the recombination continuum, whereas the increase with m/d^2 is caused by the molecular continuum. From the straight part of the line in fig. 64 we obtain (m/d^2 in mg/cm³):

$$C'_m/C'_r \approx 1. \qquad (8.\ 5.5)$$

With (8. 5.5) we are thus able to determine the ratio of the contributions of the recombination continuum and the molecular continuum as a function of m/d^2. To arrive at the ratio of the continuous energy to the energy in the lines, we proceed as follows: The total energy in the line spectrum P_{lines} may be represented by:

$$P_{\text{lines}} = C_l\, p\, T^{-1} \beta^2 R^2 \exp(-e\overline{V}/kT_{\text{eff}}), \qquad (8.\ 5.6)$$

$\overline{V}$ representing the height of the fictitious mean upper level (see II,4). The energy balance equation thus becomes:

$$\left.\begin{aligned} P - P_{\text{cond}} = {} & C_l\, p\, T^{-1} \beta^2 R^2 \exp(-e\overline{V}/kT_{\text{eff}}) + \\ & + C_r\, p\, \beta^2 R^2 \exp\{-e(V_i - \Delta V_i)/kT_{\text{eff}}\} + \\ & + C_m\, p^2 T^{-1/2} \beta^2 R^2 \exp(-eV_m/kT_{\text{eff}}), \end{aligned}\right\} \qquad (8.\ 5.7)$$

expressing the fact that the input P is equal to the loss by conduction P_{cond} together with the losses caused by radiation in the lines, in the recombination continuum and in the molecular continuum. Neglecting again the powers of T and the differences in $\overline{V}$, $(V_i - \Delta V_i)$ and V_m, representing these voltages by V^* and with $pR^2 \backsim m$, we obtain:

$$P - P_{\text{cond}} = m\,[C'_l + C'_r + C'_m\, m/d^2] \exp(-eV^*/kT_{\text{eff}}). \qquad (8.\ 5.8)$$

According to (8. 5.8) the absolute value of the energy in the continuous spectrum is:

$$P_{\text{cont}} = (P - P_{\text{cond}}) \cdot \frac{C'_r + C'_m\, m/d^2}{C'_l + C'_r + C'_m\, m/d^2}, \qquad (8.\ 5.9)$$

where the ratio C'_m/C'_r is already known from (8. 5.5).
In fig. 65 the points represent the measured intensity of the continuous spectrum plotted on an arbitrary scale as a function of m/d^2. The form of this function is determined by the ratios between C'_l, C'_m, and C'_r, and as C'_m/C'_r is already fixed by (8. 5.5) the only variable is C'_l/C'_r. The points of fig. 65 agree very well with $C'_l/C'_r = 3$ and the full line is drawn using this value.

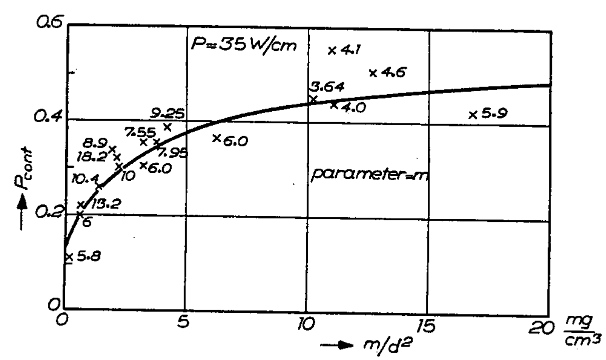

Fig. 65. The intensity of the continuous radiation, on an arbitrary scale, as a function of m/d^2 for a constant input [37]

Equation (8. 5.8) thus becomes (m/d^2 in mg/cm³):

$$P - P_{cond} = m\,C_r'\,[3 + 1 + m/d^2]\,\exp\,(-eV^*/kT_{eff}), \quad (8.\,5.10)^*$$

where the three terms between brackets represent respectively the energy in the lines, the energy in the recombination continuum and the energy in the molecular continuum.

By making relative measurements only (but supposing that the total radiation is equal to $P - P_{cond}$), we have thus obtained the absolute values of the energies in the line spectrum and the continuous spectrum. According to (8. 5.10) the continuous energy amounts to

$$P_{cont} = (P - P_{cond})\,\frac{1 + m/d^2}{4 + m/d^2}, \quad (8.\,5.11)^*$$

whereas the energy in the lines amounts to

$$P_{lines} = (P - P_{cond})\,\frac{3}{4 + m/d^2}. \quad (8.\,5.12)^*$$

We shall compare this result with the energy measurements of the U.V. standard as given in VIII,1. For this discharge $P = 22$ W/cm or $P - P_{cond} = 12$ W/cm. As $m/d^2 = 3.7/3.24 = 1.14$, we obtain from (8. 5.11) and (8. 5.12)

$$\left.\begin{aligned} P_{cont} &= 12 \times 0.415 = 5.0 \text{ W/cm} \\ P_{lines} &= 12 \times 0.58^5 = 7 \text{ W/cm.} \end{aligned}\right\} \quad (8.\,5.13)$$

and

The measured values using Rössler's figures, (see VIII,1) are:

$$\left.\begin{aligned} P_{cont} &= 17/8 = 2.1 \text{ W/cm} \\ P_{lines} &= 51/8 = 6.4 \text{ W/cm.} \end{aligned}\right\} \quad (8.\,5.14)$$

The measured total radiation of 8.5 W/cm is 71 % of the energy of 12 W/cm, which is supposed to leave the discharge as radiation (in agreement with equation (2. 3.4). According to (8. 5.13) and (8. 5.14) the radiation of 1 cm of the discharge of the U.V. standard which does not reach the measuring instrument consists of 2.9 W continuous radiation and 0.6 W line radiation.

At the end of section VIII,3 we found that the measured intensity of

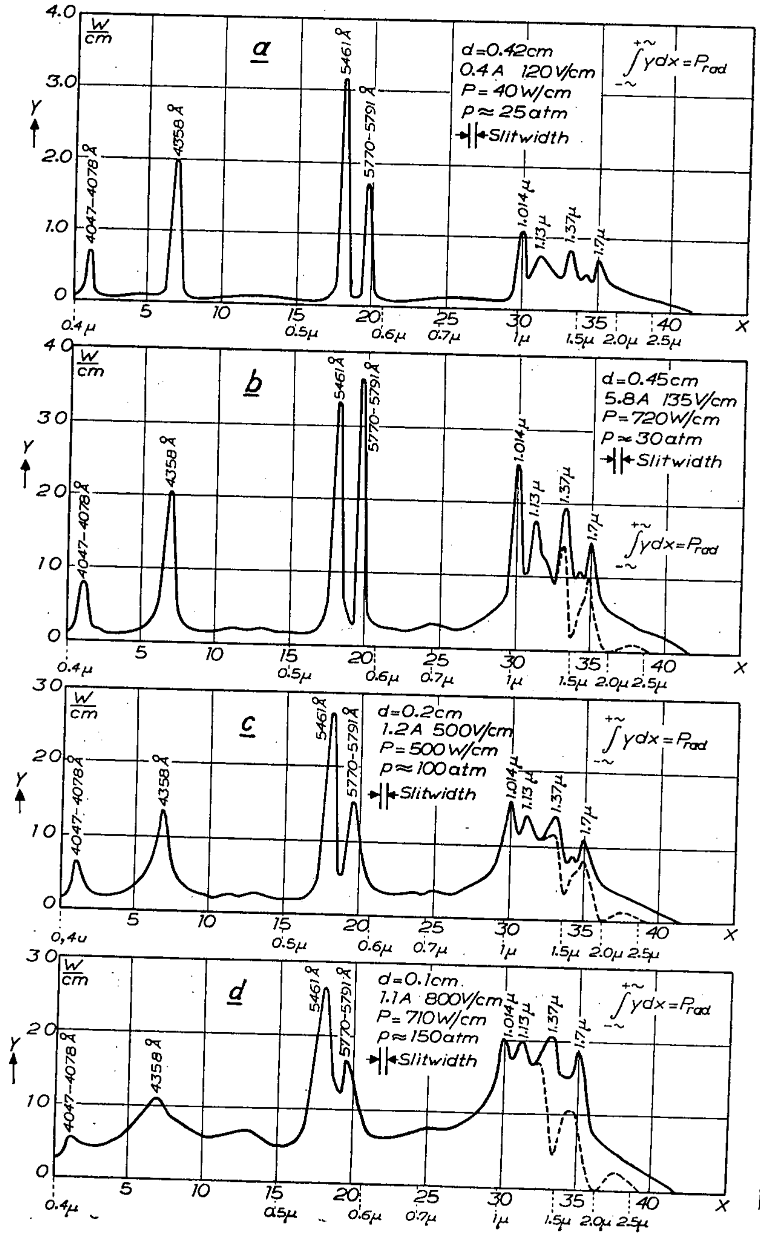

Fig. 66. Energy distribution at different diameters, pressures and loadings per cm. The curves of b, c and d belong to watercooled lamps. The broken line represents the radiation changed by the absorption of circa 0.3 mm water [23]. (The ordinate y represents the energy radiated by 1 cm of arc length, per unit of the abscis X)

the continuous spectrum was a factor 5 higher than the intensity calculated from (8. 3.1). We may now check whether the part of the continuous energy which, according to (8. 5.10), is due to recombination, is in fact in agreement with equation (8. 3.1). Since for the discharge D, m/d^2 is equal to 0.715, the recombination energy represents 58 % of the total continuous energy. The dis-

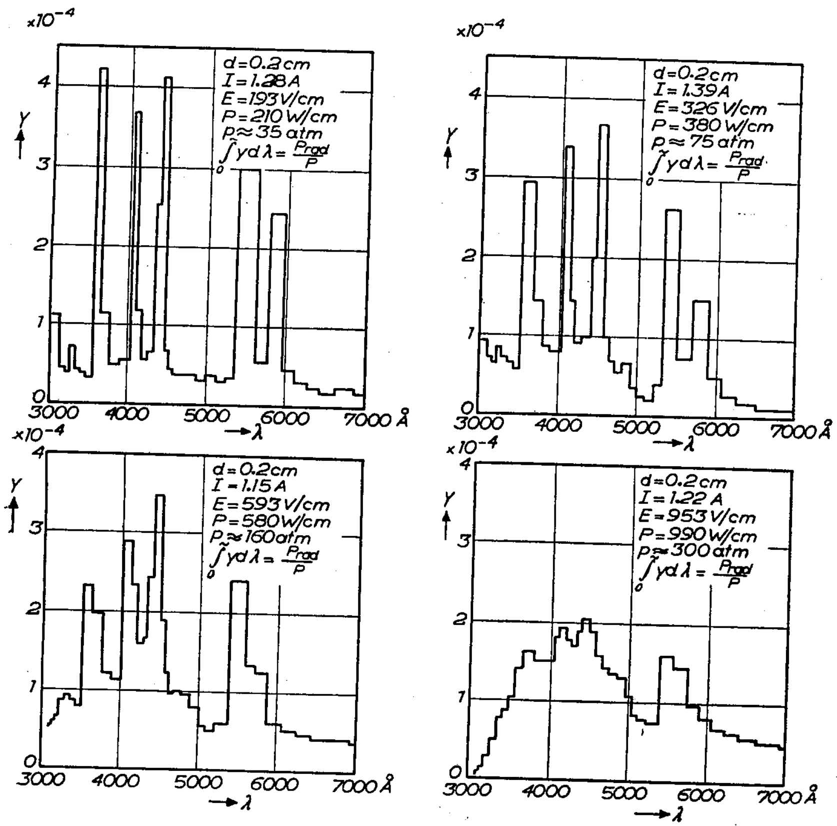

Fig. 67. Block diagrams of the energy distribution as a function of wavelength for watercooled high pressure mercury vapour lamps at approximately constant current and increasing pressure, potential gradient and input/cm [50, 96] (the ordinate y represents the radiation energy per Å per watt input)

crepancy of a factor 5 is thus reduced to a factor 3. Thus it seems that equation (8. 3.1) gives an energy value which is too small. Whether the factor of (8. 3.1) is too small, or whether there is another mechanism operative which produces an intensity of the continuous spectrum proportional to p is unknown.

In the case of the high pressure Kr and Xe discharge, equation (8. 3.1) has been used to calculate the potential gradient by supposing that the radiation of the high pressure Kr and Xe discharge is due to recombination [40]. However, in these calculations ψ enters in the result as $\psi^{1/2}$, so that a deviation of a factor $\sqrt{3}$ in the Q value is already sufficient to compensate for a factor 3 in ψ. Q being not accurately known for Kr and Xe it is not possible to decide from these experiments whether equation (8. 3.1) is correct within a factor 3.

According to (8. 5.11) the continuous spectrum contains 25 % of the total radiation at the lower pressures (of which an appreciable part may be absorbed — see (8.5.13) and (8.5.14)), whereas at very high pressure the radiation is only to be found in the continuous part of the spectrum due to the molecular radiation. The energy distributions (figs. 60, 66 and 67) indeed show that the energy in the lines approaches zero at high vapour densities.

The ultraviolet spectrum is shown in fig. 62, whereas fig. 68 shows the visible spectrum of various discharges.

As a result of the equalization of $\bar{V}$, $(V_i - \Delta V_i)$ and V_m (see equations (8.5.7) and (8.5.8)) the intensity of the continuous spectrum relative to the intensity of the lines is shown by (8. 5.11) and (8. 5.12) to be independent of the current. In practice, however, the intensity of the continuum with increasing current (d and m held constant) increases with respect to the line intensity. This is the result of the increasing temperature. Since $\bar{V}$ is smaller than both $(V_i - \Delta V_i)$ and V_m, the recombination continuum and the molecular continuum both experience this relative increase of intensity.